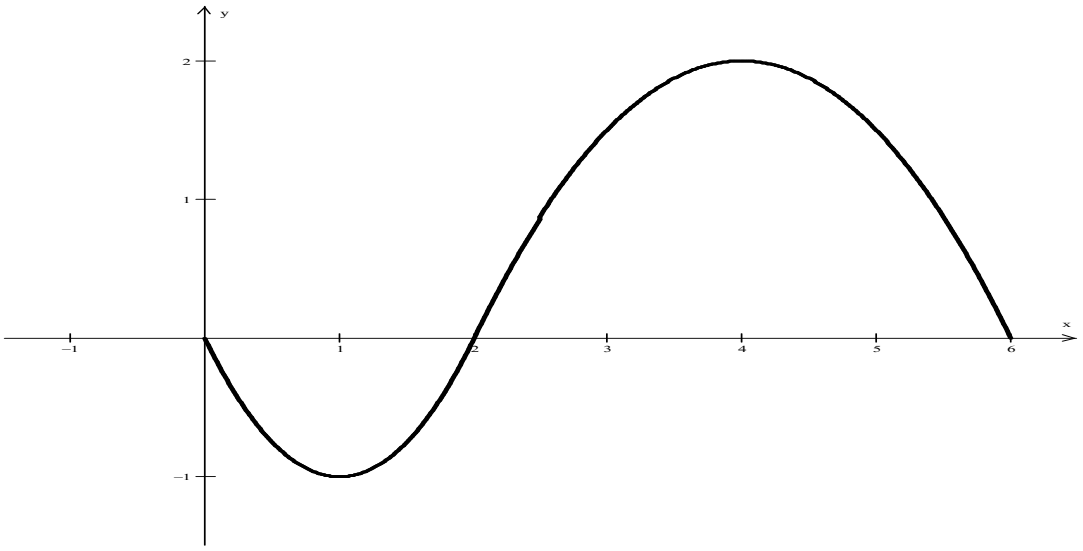
AP Calculus Derivatives Test Name:

1. The graph of $f'(x)$ is shown below. For which of the following values of x is $f(x)$ concave down?



A. $x = \frac{1}{2}$
B. $x = \frac{3}{2}$
C. $x = 2$
D. $x = \frac{5}{2}$
E. $x = 3$

2. Find the coordinates of the point where the line tangent to the parabola $y^2 = 8x$ at $(2,4)$ intersects the axis of symmetry of the parabola.

3. Chord $\overline{AB}$ of the parabola $y = x^2$ moves up from the vertex at $\frac{3}{4}\frac{units}{sec}$, always remaining parallel to the x-axis. Triangle ABC is formed by $\overline{AB}$ and the two tangents to the parabola at A and at B. The triangle increases in area as $\overline{AB}$ moves up. Find the rate at which area is increasing in $\frac{units^2}{sec}$ when $\overline{AB}$ is 4 units above the vertex.

4. Given the curve $y^2 = 150 - 10x,\ 0 \leq x \leq 15,$ find the coordinates of the point on the curve in the first quadrant that is nearest the origin.

5. The curve $y = \frac{ax+b}{(x-4)(x-1)}$ has a relative maximum at the point $(2, -1)$. Find a and b.

6. If $f(x) = e^x,\ g(x) = \sin x,$ and $h(x) = f\big(g(x)\big),$ then $h'\left(\frac{\pi}{2}\right) =$

   A. -1
   B. 0
   C. $e^{-\frac{\pi}{2}}$
   D. 1
   E. $e^{\frac{\pi}{2}}$

7. If the function $f(x)$ is such that $f(1) = 4,\ f(2) = 4,$ and $f''(x)$ exists and is positive on the closed interval $[0,4],$ then we must have

   A. $f'(1.5) = 0$
   B. $f'(1.5) > 0$
   C. $f'(3) > 0$
   D. $f'(3) < 0$
   E. None of the above

8. Which of the following functions does not satisfy the conditions necessary to apply Rolle's Theorem?

   A. $f(x) = x^3 - x$ on $[0,1]$
   B. $f(x) = \sqrt{8 - x^3}$ on $[-2,2]$
   C. $f(x) = x^{\frac{4}{3}} - 1$ on $[-1,1]$
   D. $f(x) = \frac{x^2-4}{x-3}$ on $[-2,2]$
   E. $f(x) = x^2 - 4x$ on [0,4]

9. The function $f(x) = x^3 - 4x^2 + 9x - 5$ in the vicinity of the point $(1,1)$ is

   A. Increasing at a decreasing rate
   B. Increasing at an increasing rate
   C. Decreasing at a decreasing rate
   D. Decreasing at an increasing rate
   E. Neither increasing nor decreasing

10. The line $y = 4x + 13$ is tangent to the curve $y = -2x^2 + kx + 5$, where $k < 0$. Find k.

11. Let $f(x) = \begin{cases} x + 2a, & x < 1 \\ ax^2 + 7x - 4, & x \geq 1 \end{cases}$. If a is such that $f(x)$ is continuous at $x = 1$, is $f(x)$ also differentiable at $x = 1$? Justify your answer.

12. Determine the coefficient c so that the curve $f(x) = ax^3 + bx^2 + cx + d$ has a relative minimum at $(2, -10)$ and a point of inflection at $(0,6)$.

13. The coordinates of the point where the normal to the curve of $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$ when $x = 1$ intersects the y-axis are

A. $\left(0, \frac{3}{2}\right)$
B. $\left(\frac{3}{2}, 0\right)$
C. $\left(0, \frac{13}{6}\right)$
D. $\left(\frac{13}{6}, 0\right)$
E. $\left(0, \frac{5}{3}\right)$

14. $3x^2$ and $3x^3$ are both increasing and positive for $x > 0$. How many times greater is the rate of increase of $3x^3$ than the rate of increase of $3x^2$ at $x = 4$?

A. 2
B. 4
C. 6
D. 7
E. 16

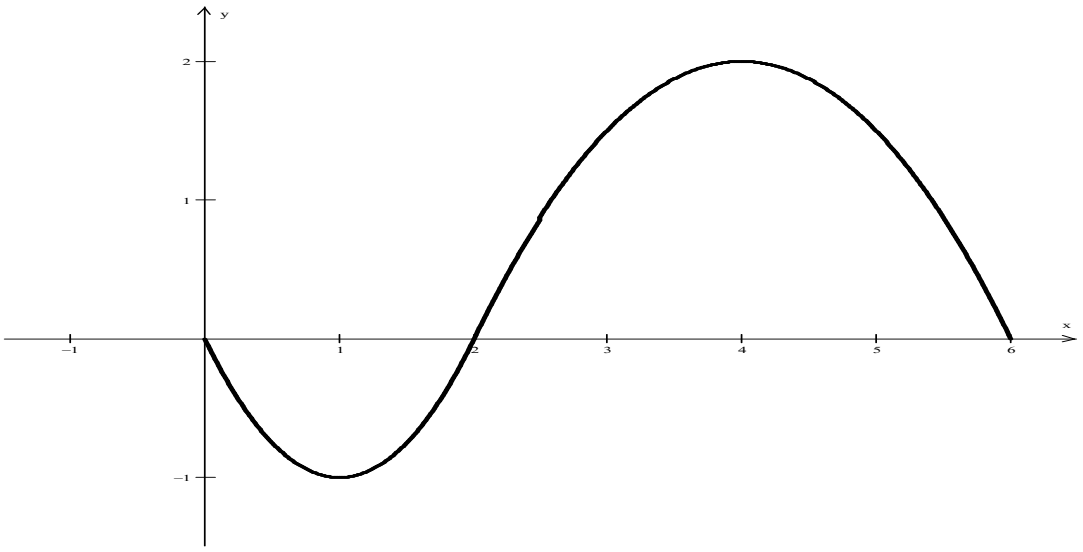
15. $y = x - \frac{k}{a^2}x^2,\ k > 0$. If a is doubled, then the maximum height of the curve

A. Remains the same
B. Is increased by a factor of 2
C. Is decreased by a factor of 2
D. Is increased by a factor of 4
E. Is increased by a factor of 4

16. Let f and g be differentiable functions, where $f(2) = 6, g(2) = 4, f'(2) = -5,\ g'(2) = -2,\ f'(4) = -3,\ g'(4) = 3.$ If $h(x) = f\big(g(x)\big),$ then $h'(2) =$

A. -20
B. -12
C. -6
D. -3
E. 6

17. The graph of $f'(x)$ is shown below. On the interval $0 \le x \le 6$, for what value of x does $f(x)$ achieve its absolute maximum value.



A. $x = 0$
B. $x = 2$
C. $x = 4$
D. $x = 6$
E. Cannot be determined

18. If $f'(x)$ exists for all x and $f(1) = 10$ and $f(8) = -4$, then, for at least one value of c in the open interval $(1,8)$, which of the following must be true?

A. $f(c) = 12$
B. $f(c) = -12$
C. $f'(c) = 2$
D. $f'(c) = -2$
E. $f(c^2) = 16$

19. What is the value of k such that the curve $y = x^3 - \frac{k}{x}$ has a point of inflection at $x = 1$?

A. $k = 2$
B. $k = -2$
C. $k = 3$
D. $k = -3$
E. None of the above

20. Find a positive value of x that satisfies the Mean Value Theorem for $f(x) = \sin x$ on the closed interval $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$.

21. $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$. If x is increasing at 4 units/sec and y is increasing at 6 units/sec, how fast is z increasing when $x = 20$ and $y = 30$?